Advanced Controller for Repetitive Operation

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Abstract: *This paper presents an innovative controller for a plant executing a repetitive work cycle with a prefixed time period. No restriction are posed on the shape of the reference variable. The controller can be very easily designed by applying the fundamentals of control theory and some very standard tools for the analysis of system dynamics. Although structure of the controller is very simple, suitable modifications improve the flexibility and the accuracy just from the first work cycle. The controller can be easily implemented by using digital techniques. The application to a mechanical system, with very relevant static friction, confirmed the high quality of the controlled plant performances.*

Key words: *repetitive control, dedicated controller, time-delay control, motion control applications, friction compensation.*

Introduction

The automation of many plants requires the execution of a repetitive work cycle and the rejection of related periodic disturbances. Dedicated control strategies, based on the learning principle [5-8], allow the improvement of the performance specifications obtainable by conventional control strategies.

This paper presents an innovative controller applied to the motion control of a mechanical system characterised by a relevant static friction. The design procedure requires the knowledge of the dominant plant dynamics and the value of the repetitive work period. The fundamentals of control theory are necessary for approaching the design procedure which can be processed on a personal computer by applying some standard tools for the analysis of dynamic systems. The controller can be easily implemented by using digital techniques. Experimental results, effected on a real mechanism, have confirmed the high quality of the controlled system performances.

Problem formulation

As above mentioned, the goal is to obtain tracking of a periodic reference variable having an undefined shape, even when relevant variations of plant behaviour occurs.

The design of the controller should be accomplished so as to obtain:

1) closed loop stability;
2) asymptotic tracking of an unknown periodic reference variable;
3) robustness of the controlled plant performances when relevant variations of plant behaviour occurs.

The design of the controller cannot be accomplished by applying the conventional design procedure developed in terms of a reference variable which has a simple canonical shape. In the application of repetitive control, the flexibility should be obtained in terms of a reference variable which cannot be approximated by only a few simple canonical signals [1] [3].

The starting point in the design of the controller will be the well-known principle according to which the tracking can be obtained by a controller which has an impulsive response with the same steady state shape as the reference variable. For example, a controller with a transfer function having a pole at the origin realises an asymptotic tracking when a step variation of the reference variable occurs.

The design of the innovative controller will be an extension of the previously mentioned principle. The attainment of the closed loop stability and the improvements in the accuracy specifications will suggest the modifications necessary for achieving a higher quality in the controlled system performances.

Controller design

As previously said, the controller should indefinitely reproduce a periodic signal of a generic shape and of a prefixed time-period T [2]. A finite delay included in a positive loop allows the attainment of this fundamental performance. Fig. 1 shows the block diagram implementation of an ideal controller. The fundamental behaviour is in fact represented but the closed loop stability and the tracking accuracy can be achieved by introducing suitable modifications.

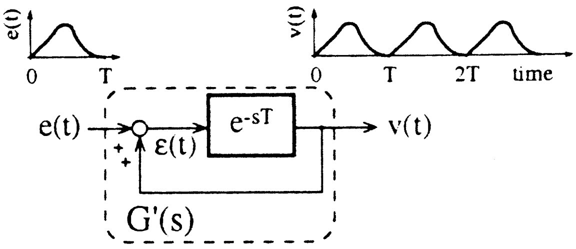


Fig. 1: Block diagram of the ideal controller.

From Fig. 1 the transfer function G'(s) of the ideal controller is easily deduced:

$$G'(s) = \frac{V(s)}{E(s)} = \frac{e^{-sT}}{1 - e^{-sT}} \qquad (1)$$

The block diagram of the controlled plant, using the above illustrated controller, is shown in Fig. 2.

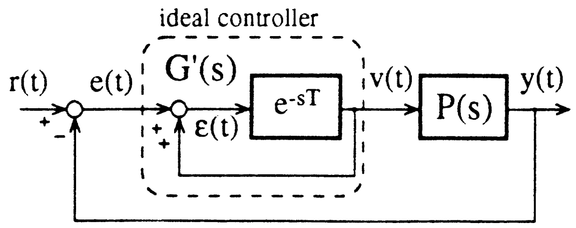


Fig. 2: Block diagram of the controlled plant equipped by the ideal controller.

A more direct verification of the closed loop stability condition can be effected by taking into account the transfer function $W_e(s)$ between the reference variable R(s) and the error variable E(s). This latter, in terms of the controller and plant dynamics results:

$$E(s) = R(s) - Y(s) = \qquad (2)$$
$$R(s) - P(s)\, e^{-sT}[\, V(s) + E(s)]$$

Therefore the transfer function $W_e'(s)$ is:

$$W_e'(s) = \frac{E(s)}{R(s)} = \frac{1 - e^{-sT}}{1 - e^{-sT}[1-P(s)]} \qquad (3)$$

It can be represented by the block diagram shown in Fig. 3 that is quite equivalent to that of Fig. 2 for the verification of the closed loop stability.

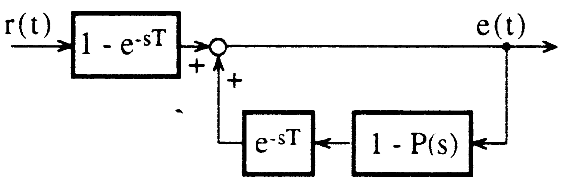


Fig. 3: Block diagram of the controlled plant for the closed loop stability verification.

The closed loop stability is verified if the magnitude of the loop transfer function, i.e. e^{-sT} [1 - P(s)], is lower than 1 in the whole frequency domain. Since the transfer function of a time delay, i.e. e^{-sT}, has an magnitude equal to 1 in the frequency domain, the closed loop stability condition can be formulated as follows:

$$\| 1 - P(s) \|_{\infty} < 1 \qquad (4)$$

Since the majority of plants have a low pass characteristic, i.e. P(jw) = 0 for wÆ •, the previously suggested ideal controller cannot directly be applied because the stability condition is not verified in whole frequency domain. A slight modification should be consequently introduced in the structure of the above proposed ideal controller.

A low pass filter Q(s) introduced before the time delay allows one to verify the stability condition. The modified controller therefore has the structure shown in Fig. 4, which represents the block diagram of the controlled plant.

The stability condition, given by Eq. (5), should be modified to take into account the transfer function of the low pass filter. The transfer function between the error variable E(s) and the reference variable R(s) becomes:

$$W_e''(s) = \frac{E(s)}{R(s)} = \frac{1 - e^{-sT}\, Q(s)}{1 - e^{-sT}\, Q(s)\, [1 - P(s)]} \qquad (5)$$

and consequently the stability condition results:

$$\| Q(s)\, [1 - P(s)] \|_{\infty} < 1 \qquad (6)$$

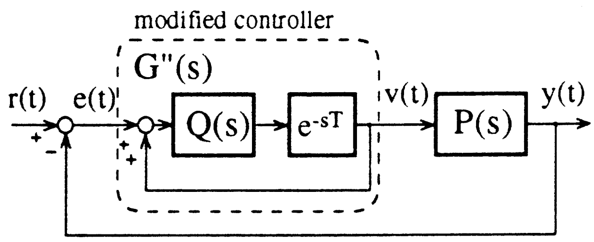


Fig. 4: Block diagram of the controlled plant implementing the modified controller.

Although the low-pass filter satisfies the stability condition has the drawback to reduce the dynamics of the tracking due to its influence on higher harmonics of the periodic reference variable. This drawback can be removed by overlapping a signal proportional to the above defined error variable, i.e. e(t), to the low pass filter output.

Such a modification recovers the accuracy in the tracking of the controlled variable during the steady state operation, but it has no effect on the evolution of the first work cycle. At this time, due to the time-delay operating in the direct loop; the feedback signal is broken and a free evolution of the plant occurs. The error in the tracking could therefore be relevant. It can be removed by introducing a further modification in the controller structure. It consists of applying a signal proportional to the previously defined error variable directly to the input of the plant. By this further modification the feedback loop is never interrupted.

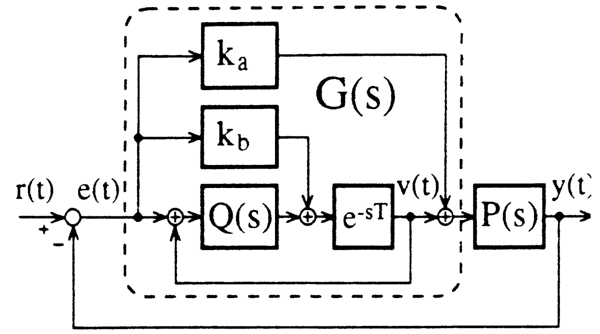


Fig. 5: Block diagram of the controlled plant implementing the innovative controller.

Fig. 5 shows the block diagram of the controller implemented with the previously mentioned modifications. It realises the innovative controller.

The above mentioned modification in the structure of the controller requires a new formulation of the stability conditions. The transfer function between the error variable E(s) and the reference variable R(s) becomes:

$$W_e(s) = \frac{E(s)}{R(s)} = \frac{\dfrac{1 - e^{-sT} Q(s)}{1 + k_a P(s)}}{1 - e^{-sT} Q(s) \left[1 - \dfrac{[k_b + Q(s)] P(s)}{Q(s) [1 + k_a P(s)]} \right]} \tag{7}$$

which corresponds to the block diagram of Fig. 6

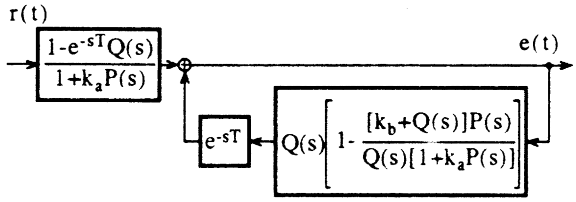


Fig. 6: Block diagram of the controlled plant for the closed loop stability verification.

The stability of the closed loop system implies:

- the stability of the open loop system transfer function [4], i.e.:

$$P^*(s) = \frac{[k_b + Q(s)] P(s)}{Q(s) [1 + k_a P(s)]} \tag{8}$$

-the fulfilment of the following condition:

$$\left\| Q(s) \left[1 - \frac{[k_b + Q(s)] P(s)}{Q(s) [1 + k_a P(s)]} \right] \right\|_\infty < 1 \tag{9}$$

The design of the controller starts from above mentioned stability conditions. The fulfilment of the first constraint fixes the gain k_a and k_b, whereas by the fulfilment of the second constraint, the transfer function Q(s) of the low pass filter is worked out.

The gains k_a and k_b should be adjusted so as to attain the desired accuracy of the tracking in correspondence of the steady state operation and of the first work period.

The design procedure can be synthesised as follows. The transfer function Q(s) is firstly designed so as to satisfy the stability condition given by Eq.(4) and to save the magnitude of the fundamental harmonics of the reference variable.

Once the gain k_b is fixed to zero, the gain k_a is chosen so as to obtain the wider bandwidth being the following stability condition satisfied:

$$|Q(j\omega)| < \left| \frac{1 + k_a P(j\omega)}{1 + (k_a - 1) P(j\omega)} \right| \tag{10}$$

Finally, the gain k_b is chosen so as to extend the frequency range in which the module of the transfer function $W_e(s)$ has a limited magnitude, holding the stability condition.

If the procedure fails, it is repeated by updating the transfer function Q(s).

Experimental results

The innovative controller has been applied to a mechanical device driven by a DC servomotor. The command variable is the mean value of the voltage applied to the motor by a PWM power supply. The motor speed is measured on the motor shaft by a tachometer, and the mechanical load can be represented by an inertia and a viscous friction. A very relevant static friction acts on the motor shaft as it is deduced by the voltage-speed static characteristic, shown in Fig. 7. This latter evidences the limited range of the motor voltage, i.e. command variable, due to the power amplifier saturation.

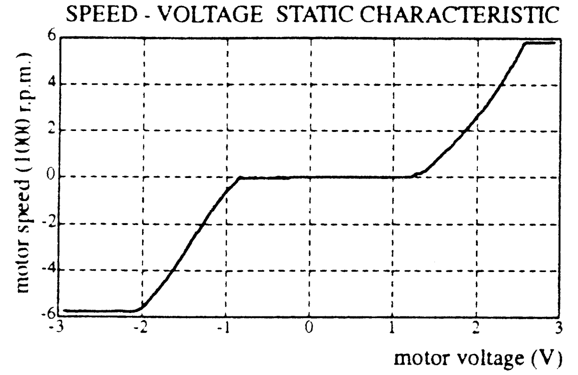


Fig. 7: Static characteristic of the motor supplied by the PWM power amplifier.

In the linear range of the static characteristic, the dynamic behaviour of the motor is described by following transfer function:

$$P(s) = \frac{1.6}{s + 1.7} \qquad (11)$$

which has been derived from experimental tests. The input signal was obtained by superimposing a pseudo binary random sequence [9] to a motor voltage fixed in the middle range of the static characteristic linearity. The output variable was obtained by the tachometer. The input-output variables has been processed by applying some programs from the Matlab Identification Toolbox.

The performance specification of the repetitive control implied the maximum accuracy in the tracking of a speed reference variable characterised by a triangular shape. The periodicity was fixed at 5 seconds whereas the amplitude was ranging from 0 to 3000 r.p.m.

The design of the controller was carried out by using the Simulink tools. The most convenient situation in the tracking accuracy was found when the following transfer function of the low-pass filter has been used:

$$Q(s) = \frac{1}{1 + .05\,s} \qquad (12)$$

when the gains k_a and k_b were respectively equal to 20 and 10.

The experimental tests were carried out by using the laboratory equipment shown in Fig. 8. The controller was implemented on DSP32C AT&T board included in the personal computer.

Fig. 9 illustrates the results related to the first six periods. The reference variable and the controlled one are illustrated in Fig. 9a) whereas the difference between these two variables in Fig. 9b). This figure shows that high accuracy is attained from the first few cycles alone.

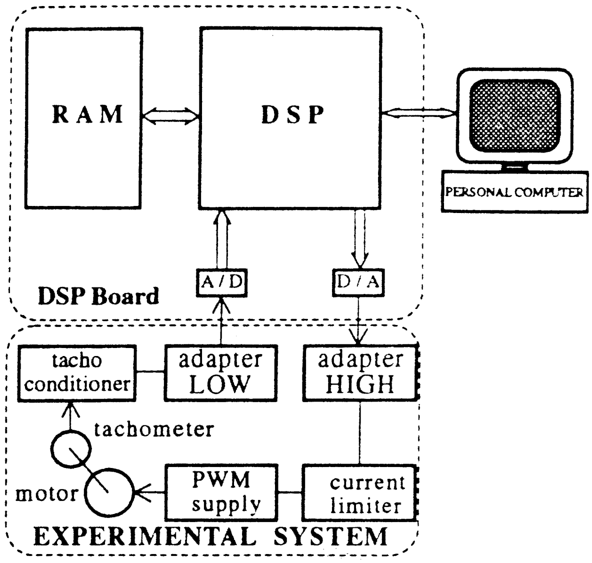
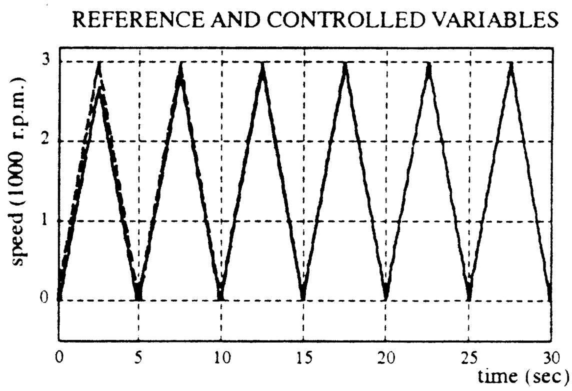


Fig. 8: Experimental equipment



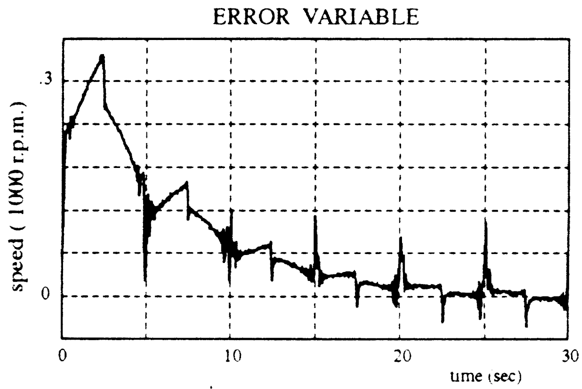


Fig. 9: Controlled system behaviour in correspondence of the first periods.

Fig. 10 illustrates the same variables and the voltage command at the 39^{th} and 40^{th} period when the steady state operation is surely attained. The high accuracy is held just in less favourable conditions corresponding to the abrupt change of the slope in the reference variable.

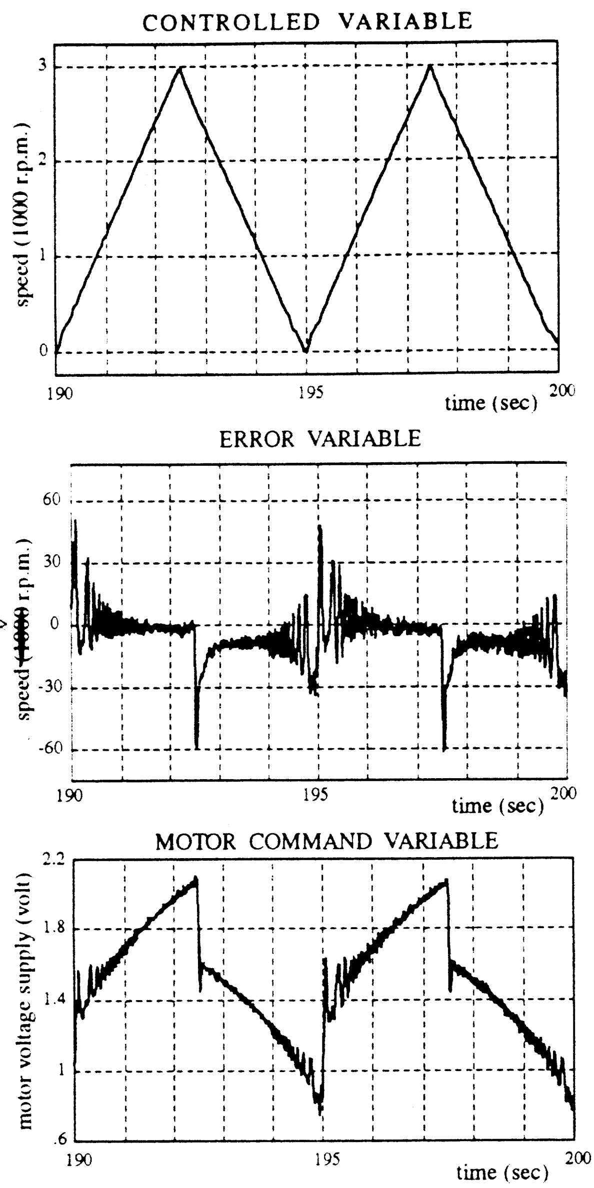


Fig. 10: System behaviour in the 39th and 40th period.

The figures also shows the satisfactory behaviour of the controlled system in spite of the relevant nonlinearities due the static friction and the power supply saturation.

Conclusions

The tracking of a repetitive work cycle is a typical operating condition in automation. In general, it requeses good flexibility, high accuracy and high productivity of the controlled plant. All these specifications depend on the selection of the controller and the drive. This latter should perform more and more as an instantaneous and linear torque generator to improve the performance specifications by means of an innovative controller. The higher cost of these drives is made up for by smaller size, higher instantaneous overload, and smaller energy dissipation.

The proposed innovative controller is based on the variable size shift register and can be easily implemented by digital techniques. Once the sampling period has been fixed, the size of the shift register depends only on the duration of the work cycle. The low pass filter and the other operations do not require a high speed digital device.

The design of the controller involves the selection of the low-pass filter parameters and the gains of the proportional actions bypassing the low-pass filter and the time delay. All these values are more efficiently ffound by a simulation approach in the continuous time domain. Dedicated toolboxes make this approach easier, as well as the transfer of the results in the discrete time domain as requested by the digital implementation.

The structure of the innovative controller is very general, but the selection of parameters requires the knowledge of the plant dynamics. It can be effected by a simulation procedure and not by heuristic approaches. This situation is typical in design of all the controller which allows the attainment of higher performance specifications.

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